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Revisiting the D1/D5 system or bubbling in AdS$_3$

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Abstract: In this article we study the relation between the bubbling construction and the Mathur's microscopic solutions for the D1/D5 system. We have found that the regular near horizon D1/D5 system (after appropriated constraints are imposed) contains all the bubbling regular solutions. Then, we show that the features of this system are rather different from the bubbling in $AdS_5 \times S^5$, since the perimeter and not the area plays a key role. After setting the main dictionary between the two approaches, we investigate on extensions to non-regular solutions like conical defects and/or naked singular solutions. In particular, among the latter metrics, closed time-like curves are found together with a chronology protection mechanism enforced by the AdS/CFT duality.

Keywords: String Duality, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

String theory is the most promising candidate to accommodate all the fundamental forces of the universe, including gravity. Unfortunately, our understanding of this theory is far from complete. In particular, how quantum gravity and the standard model are realized within string theory is still elusive.

However, we do not need to solve in full string theory to learn important lessons: in some cases, even simple toy models produce amazing physical outputs. This is the case of the AdS/CFT conjecture, where we can study closed string theory (and therefore quantum gravity) as dual to super Yang-Mills theory (SYM). Actually what we really mean is that within this framework, we can concentrate on a sector of the theory where a lot of control is achieved but physical relevance is still present.

Among the many different possibilities, our studies are motivated by recent works on 1/2 BPS sectors of near horizon geometries sourced by a stack of N D3-branes [1–4]. In this case, the *bubbling construction* consists in looking for solutions of type IIB supergravity theory, where only the metric and the self dual Ramond-Ramond 5-form are excited. Furthermore, it is required to have regular solutions with an SO(4) $\times$ SO(4) symmetry and at least 1/2 BPS supersymmetry. Once the above solutions are found, by means of the AdS/CFT duality, the corresponding dual operators in N=4 SYM theory are identified. One of the most interesting outcomes of the above studies is the simplicity of the dual field theory, a matrix quantum mechanics, that gives the possibility to obtain a deeper understanding on fundamental issues of quantum gravity: for example the role of closed time-like curves (CTC) [5], the study of black hole thermodynamics [6] and the appearance of stretched horizons [7] probing black hole entropy laws, and even the structure of the quantum phase space [8, 9].

Although there is a growing amount of literature on this subject, we noticed that the other superconformal cases, M2, M5, D1/D5, are less understood and therefore they deserve some attention. Following the above reasoning, we study the AdS/CFT conjecture for the D1/D5 case, focusing on the simplest 1/4 BPS sector (from the ten dimensional point of view) with an $SO(2) \times SO(2)$ symmetry, that is known as *bubbling in* AdS_3.

Bubbling in AdS_3 has already been studied, to the best of our knowledge, in three papers [10 − 12]. First we have two papers produced by Liu et al, that consider from first principles mainly the supergravity side, leaving open the relation with the CFT theory. Secondly, the work of Martelli and Morales, that uses an already known family of solutions to obtain the supergravity equations. Then, they make a conjecture relating the regular bubbling solutions to the known regular solutions of the D1/D5 system constructed by the group of Mathur [13 − 15]. We believe that there are still many important points that need to be clarified. In particular, before deeper studies can be done, we need to set the basics of the AdS/CFT dictionary and to get a better understanding on the relation between the bubbling supergravity solutions and the related dual CFT operators, including a study of regularity conditions and of the way other non regular geometries could appear in this framework. The scope of the present work is precisely to add on this direction.

The paper is organized as follows: in section 2 we review the form of the bubbling ansatz from [12], paying particular attention to the field equations and their general solution in terms of two different types of sources or boundary conditions on a two dimensional plane. Then, adding the regularity requirement, we are able to reduce the two types of sources into a single one, constrained to live on a monodimensional curve. We compute the total flux of the solutions finding that it comes in terms of a line integral on the source and it is proportional to its length. In section 3 we review the D1/D5 supergravity solution, together with the basic input coming from the CFT theory. Then, we show that it is straightforward to constrain the D1/D5 solutions to include the solutions discussed in the previous section, realizing the conjectured relation between the two approaches presented in [11]. At this point, the basics of the AdS/CFT dictionary for the bubbling construction are given. In section 4 we show how non-regular solutions can be included into this duality study by relaxing the regularity conditions, in order to enlarge the family of solutions, obtaining conical singularities, Aichelburgh-Sexl and naked singularities. Some of these metrics have a well defined dual operator, and some don't. In particular, we have found solutions with CTC that nevertheless seem to have no counterpart in the D1/D5 system, realizing a sort of chronology protection mechanism enforced by string theory. In section 5 we comment on the CFT dual description and the possible role of the Liouville theory in the duality, together with a short discussion on future work.

It is important to highlight that along this article we will be always working within the minimal supergravity framework. Hence, we only excite the metric g and the 3-form field strength $H_{(3)}$, to avoid further complications. Nevertheless, inclusion of tensor multiplets should not be much more difficult and is left for future works. We also point out that, although this last reduction seems to exclude giant graviton configurations (since they have been identified with supergravity solutions including non-trivial dilaton field [13]), we still find states that look very much like a giant graviton, at least from the bubbling point of view. More on this can be found in section 5.

2. Bubbling ansatz for AdS_3

We start this section by reviewing some known facts about bubbling in six dimensional models (see [11, 12] for a derivation of the supergravity ansatz). The working hypothesis is to look for the simplest states in the D1/D5 system, identifying their quantum numbers and symmetries to translate them into isometries on the supergravity side, giving form to the corresponding ansatz.

In short, for the near horizon D1/D5 system, the spectrum of chiral primaries is classified by the conformal dimensions $(h, \bar{h})$ and the R-charges $(j, \bar{j})$, related to the symmetries $\mathrm{SO}(2,2) \times \mathrm{SO}(4) \sim \mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SL}(2,\mathbb{R})_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Here, we are interested in $N = (1,0)$ minimal six dimensional supergravity, that is universal for Kaluza-Klein reductions from ten dimensions, either on T^4 or on $K3$. The simplest family of states in this setting is given by $h = \bar{h}$ and $j = \bar{j}$ (see [16 − 18] for studies on chiral primaries).

The corresponding supergravity ansatz is defined by the above symmetries and the nature of the chiral states under study. The general form of the solution is worked out from first principles in [10, 12] or deduced from previously known results, in [11]. In both cases the metric is given by[1]

$$\begin{aligned} ds_6^2 &= -h^{-2}\left(dt+V\right)^2 + h^2\left(dy^2+\delta_{ij}\,dx_i dx^i\right) + y\left(e^G d\theta_1^2 + e^{-G}\left(d\theta_2+\chi d\theta_1\right)^2\right), \\ e^{-G} &= h^2 y + (h^2 y)^{-1}\left(z-\frac{1}{2}\right)^2, \qquad \chi = -e^G\left[h^2 y + (h^2 y)^{-1}\left(z^2-\frac{1}{4}\right)\right], \\ dV &= -\frac{1}{y} *_3 dz\,. \end{aligned} \tag{2.1}$$

where $i = 1, 2$ and $*_3$ is the three dimensional Hodge dual of flat metric in the space directions (y, x^1, x^2). The self-dual 3-form field strength $H_{(3)}$ is written in terms of a 2-form $\tilde{F}_{(2)}$ and the four dimensional Hodge dual $*_4$ of the metric in the (t, y, x^1, x^2) directions, as follows:

$$\begin{aligned} H_{(3)} &= -\frac{1}{2}[F_{(2)} \wedge d\theta_1 + \tilde{F}_{(2)} \wedge (d\theta_2 + \chi d\theta_1)]\,, \qquad F_2 = e^G *_4 \tilde{F}_2\,, \\ \tilde{F}_{(2)} &= -2\left[d\left((1+\chi)\,ye^{-G}\right) \wedge (dt+V) - h^2 e^{-G} *_3 \left(d\left(ye^G\right) + (1+\chi)\,ye^{-G} d\chi\right)\right]. \end{aligned}$$

At last, we have that h^2 and z are constrained by the equations

$$d[*_3 y d(h^2)] = 0 \qquad d[*_3 \frac{1}{y} dz] = 0\,.$$

Therefore, the whole solution is defined by these two independent functions h^2 and z, obeying second order differential equations. Expanding these last two equations, we observe that they can be understood as Laplace equations in four and six dimensional auxiliary spaces,

$$y\,\Delta_{(4)}(h^2) = 0\,, \qquad y\,\Delta_{(6)}\left(\frac{z}{y^2}\right) = 0\,.$$

[1] In this work we will always write the six dimensional metric, and not the corresponding ten dimensional metric, in order to include the possibility to work with either T^4 or $K3$.

Hence, the general solution can be written in terms of the Green functions with sources at the plane $y=0$,

$$h^2(x_1,x_2,y)=\int_{\mathbb{R}^2}\frac{\rho(x_1',x_2')dx_1'dx_2'}{(x-x')^2+y^2}\,,\qquad z(x_1,x_2,y)=\frac{y^2}{\pi}\int_{\mathbb{R}^2}\frac{z(x_1',x_2')dx_1'dx_2'}{[(x-x')^2+y^2]^2}\,.$$

Notice that we have introduced $\rho(x^1,x^2)$ as the source only for h^2, since for z the source coincides with its value at $y=0$ i.e. $z(0,x^1,x^2)\equiv z(x^1,x^2)$.

Up to this point, the above solutions solve the bubbling ansatz part related to the symmetries. It is still needed to impose regularity conditions to finish the work. In previous papers, it was conjectured that such a condition would connect these solutions to the known solutions of D1/D5 system characterized by the profile $\mathbf{F}$ of the corresponding winding string. This is a natural conjecture since we are just describing the same system from a different perspective and, therefore, the two approaches have to be connected. In what follows we provide such analysis.

Regularity condition

Following the analysis done in the LLM paper [4], we study how the regularity condition imposes constraints upon our metrics. Due to the form of the solution, the possible conflicting regions are at the $y=0$ plane. In fact, we can see that in order to have smooth geometries as we approach the source plane, one or both radii of the circles associated to θ_1 and θ_2,

$$R_1^2=y(e^G+e^{-G}\chi^2)\qquad\text{and}\qquad R_2^2=ye^{-G}\,,$$

should mix with the y coordinate and, also, h^2 or its inverse has to be regular.

We first consider the case where R_1 remains constant and R_2 recombines to y. In this case, after expanding all the relevant fields in y, we arrive to the following result

$$\begin{aligned}
z&=\frac{1}{2}+z_2y^2+O(y^3)\\
h^2&=h_0^2+h_1^2y+O(y^2)\\
e^{-G}&=h_0^2y+h_1^2y^2+O(y^3)\\
\chi&=-\frac{1}{h_0^2}\left(h_0^2+\frac{z_2}{h_0^2}\right)+O(y)
\end{aligned}$$

where $f_n=\frac{d^n}{dy^n}f|_{y=0}$ and f is any of the involved functions. The other possibility where we have a vanishing R_1 and a constant R_2 gives

$$\begin{aligned}
z&=-\frac{1}{2}+z_2y^2+O(y^3)\\
h^2&=h_0^2+h_1^2y+O(y^2)\\
e^{-G}&=\frac{1}{h_0^2y}-\frac{h_1^2}{h_0^4}+\left(h_0^2-2\frac{z_2}{h_0^2}\right)+O(y^2)\\
\chi&=-h_0^2\left(h_0^2+\frac{z_2}{h_o^2}\right)y^2+O(y^3)
\end{aligned}$$

Notice that the above set of equations tells us that z is a constant on the $y = 0$ plane, with values $(1/2, -1/2)$ only. Also, notice that χ has a rather different behaviour on the two regions of the plane, and that up to now, h^2 is unconstrained. We can use z to define two different regions on the plane, region (I) where $z = 1/2$ and region (II) where $z = -1/2$. Since we are in a two dimensional hypersurface, the frontier has to be a curve **C**.

To complete the regularity analysis, we have to probe the vicinity of the boundary of the two regions (I) and (II) or, if you prefer, the neighbourhood of the curve **C**. Basically, we need both radii to smoothly combine with y at this locus (as occurs in the bubbling solutions for the D3-brane case, that results in the pp-wave solution).

Given a general curve **C**, we use that whatever shape it assumes, we can expand in term of the exterior curvature of **C** and that locally we can always find adapted coordinates where x^1 is perpendicular to the boundary and x^2 is parallel. Then, we change coordinates to following pp-wave alike coordinates

$$y = r_1 r_2 \,, \qquad x_2 = \frac{1}{2}(r_1^2 - r_2^2) \tag{2.2}$$

and take the limit $y \to 0$ asking for a resulting smooth geometry. After some algebra, it is not difficult to see that in order to achieve regularity,

$$h^2 = \frac{1}{2x^2}(1 + O(y^2/x^2)) \,.$$

Now, this is exactly the respond of h^2 to a source with support along the curve C, and constant value $1/2\pi$ i.e.

$$\rho(\vec{x}) = \frac{1}{2\pi} \int_C \delta(\vec{x} - \vec{c}(\gamma)) \,.$$

In fact, it can be checked that the above density produces the well known examples of $AdS_3 \times S^3$ and pp-wave in six dimensions, where circular profiles and infinite straight line are used respectively.

Therefore, we have arrived to the final form that completely defines the source for regular bubbling solutions. Simply replace the boundary constraint behaviour found before to obtain[2]

$$z(\vec{x}, y) = \frac{1}{2\pi} \oint_C \frac{\vec{n}(\vec{x} - \vec{c})}{(\vec{x} - \vec{c})^2 + y^2} + \beta \,, \qquad h^2(\vec{x}, y) = \frac{1}{2\pi} \oint_C \frac{1}{(\vec{x} - \vec{c})^2 + y^2} \,, \tag{2.3}$$

where β is related to the boundary behaviour at infinity, **C** is a general curve dividing the plane expanded by (x^1, x^2) into two regions and $\vec{n}$ is a unit normal vector pointing to the region where $z = -1/2$. Notice that the integrals are re-parametrization invariant, as we should expect for a geometrical solution in gravity.

Hence, we have seen how the two different initial functions appearing on the ansatz, become related via the same boundary condition, that comes in terms of a closed curve. Here the bubbling is realized through changing the shape of the curve. Notice that there is no reason to have a single connected curve, and that in general we will have disconnected closed curves as sources for h^2 and z.

[2]Where z is reduced to a line integral using Stokes theorem.

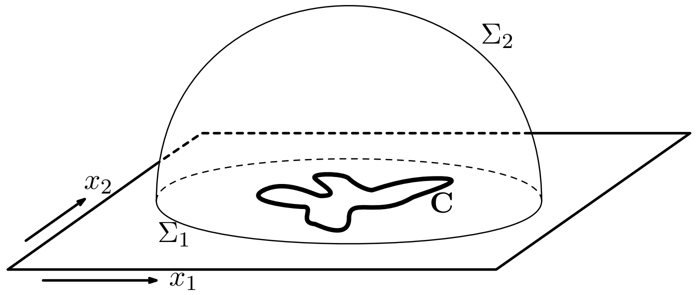

Figure 1: The flux of $H_{(3)}$ is calculated using the three dimensional hypersurface Σ_3, constructed by fibering the correspondingly not contracted S^1 with the surface Σ_2.

Next, we compute the flux f of the 3-form $H_{(3)}$ on the above solutions. Basically, we choose the three dimensional hypersurface as follows: define a two dimensional surface Σ_2 on (y, x^1, x^2) such that at $y = 0$ ends on a closed non-intersecting curve Σ_1 that encloses the curve $\mathbf{C}$, defining a disc D_2 containing $\mathbf{C}$. Then, define Σ_3 as the fibration of $\Sigma_2 \times S^1$, where S^1 is the circle not contracting to zero size on Σ_1 (see figure 1). Computation of f gives

$$\begin{aligned} f &= \int_\Sigma H_{(3)} = -\pi \int_{\Sigma_2} \tilde{F}_{(2)} = 2\pi \int_{\Sigma_2} *_3 y d(h^2) \\ &= 4\pi^2 \int_{D_2} \rho(x_1, x_2) dx_1 dx_2 \\ &= 2\pi \, \mathcal{L} \, . \end{aligned}$$

where $\mathcal{L}$ is the length of the curve $\mathbf{C}$.

Let us now summarize what we have found up to now. First, the bubbling ansatz comes in terms of two independent functions (h^2, z), sourced by independent charge distributions on the plane expanded by (x^1, x^2). Secondly, once we require regularity, the sources get identified, in terms of single distributions on a closed curve $\mathbf{C}$. The total flux f of the above solutions is proportional to the length of the curve. Based on the above,

we define bubbling on $AdS_3 \times S^3$ as all the above regular solutions, constrained to have the same flux, i.e. the same length but with arbitrary number of disconnected parts of any shape.

Notice that the above construction is similar, but different to the bubbling on $AdS_5 \times S^5$, where fixing the flux was equivalent to fix the area of the drop. Here, is the length what matters!

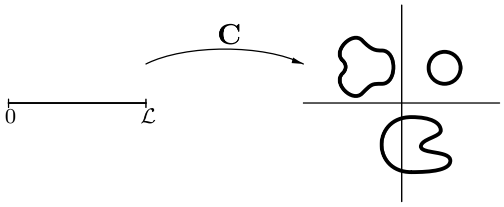

Figure 2: Generic bubbling profile: fixed the length $\mathcal{L}$, we can draw any set of disconnected closed curves.

3. D1/D5 inputs into bubbling

In this section, we study the relation between the above bubbling family of solutions and the well known D1/D5 solutions found by Mathur et al [13]. The idea is to set the dictionary to the dual CFT theory. To make the comparison simpler it is convenient to rewrite the metric (2.1) in terms of new angular variables (α, ϕ) as

$$\begin{aligned} ds_6^2 &= -h^{-2}\left[(dt+V)^2-(d\alpha+B)^2\right]+h^2\left(dy^2+y^2d\phi^2+\delta_{ij}\,dx^idx^j\right), \\ dB &= -*_4\,dV\,, \qquad\qquad B = zd\phi\,, \\ \alpha &= \frac{1}{2}(\theta_1+\theta_2)\,, \qquad \phi=(\theta_1-\theta_2)\,, \end{aligned} \tag{3.1}$$

where, in this section, we only show the metric for brevity, and have eliminated (G, χ) in terms of (h^2, z). Also, $*_4$ is the four dimensional Hodge dual, acting on $\{y, \phi, x^1, x^2\}$.

Now we turn our attention to the general near horizon metrics of the D1/D5 system (see for example [19])

$$\begin{aligned} ds^2 &= (f_1f_5)^{-\frac{1}{2}}\left[-(dt-A_Idx^I)^2+(d\alpha+B_Idx^I)^2\right]-(f_1f_5)^{\frac{1}{2}}(\delta_{IJ}dx^Idx^J)\,, \\ dB &= -*_4\,dA\,, \\ e^{2\Phi} &= f_1f_5^{-1} \end{aligned} \tag{3.2}$$

with Hodge dual $*_4$ acting on $I = 1, 2, 3, 4$ and

$$f_5=\frac{Q_5}{l}\int_0^l\frac{dv}{|\vec{x}-\vec{F}|^2}\,, \qquad f_1=\frac{Q_5}{l}\int_0^l\frac{|\dot{\vec{F}}|^2\,dv}{|\vec{x}-\vec{F}|^2}\,, \qquad A_i=-\frac{Q_5}{l}\int_0^l\frac{\dot{\vec{F}}_i\,dv}{|\vec{x}-\vec{F}|^2}\,.$$

$\vec{F}$ is a vector field that describes the embedding of the closed curve $\mathbf{F}$ along the x^I space directions, $\dot{\vec{F}}$ is the derivative of $\vec{F}$ respect to the parameter v. In this parametrization we have that $v = (0, l)$ and $l = 2\pi R' Q_5$ where Q_5 is the D5-brane charge, R' is the radius of the U-dual compact S^1 (that here is redefined to be 1), with angular variable α. Notice that these solutions are not invariant if we change the parametrization of the curve $\mathbf{F}$, since there is physical content on the above. The D1-brane charge Q_1 and the angular momentum J_{IJ} are given by[3]

$$Q_1=\frac{Q_5}{l}\int_0^l|\dot{\vec{F}}|^2\,dv\,, \qquad J_{IJ}=\frac{Q_5}{l}\int_0^l\left(F_I\dot{F}_J-F_J\dot{F}_I\right)\,dv\,.$$

The above solutions correspond to semiclassical configurations of the D1/D5 system where $\mathbf{F}$ is related to the profile of the U-dual bound state of Q_5 fundamental strings and Q_1 units of momentum. In [19] regularity conditions for this family of solutions were studied, finding that $|\dot{\vec{F}}|$ has to be different from zero, and $\mathbf{F}$ not self-intersecting. Therefore, to make contact with the bubbling solutions of (3.1), we restrict them as follows:

- $|\dot{\vec{F}}| = constant \neq 0\,,$
- $\vec{F}$ is constrained to live in two dimensions only,
- $\{x^I\} = \{y, \phi, x^2, x^1\}\,,$

[3]The angular momentum is computed in the full solution, before the near horizon limit is considered.

- $h^2 = (f_1 f_5)^{1/2}$, $V_I = -A_I$,
- we identify the curves **C** and **F** on a fixed parametrization.

The first point is necessary to work with minimal supergravity and regular solutions, the second reduces the solutions to the case where $j = \bar{j}$, and the curve **F** to be embedded into a plane. The third and fourth are just the obvious identifications of coordinates and functions, while the last identifies the curves. It is important to not only identify the curves, but to fix a parametrization once and for all, due to the fact that 3.2 depends explicitly on this parameter.

AdS/CFT dictionary

At this point, we are ready to derive the dictionary between gravity solutions and CFT states. Let us begin with some basic facts and definitions. First of all, since $|\dot{\vec{F}}|$ is constant, we get that $|\dot{\vec{F}}| = \sqrt{Q_1/Q_5}$. Then, we compute the total flux to obtain that $f = 4\pi^2\sqrt{Q_1 Q_5}$. Therefore, we learn that the parametrization of **C** should be identical to the parametrization of **F**. Hence, in gravity, the rapidity we circulate on the curve fixes the density of D1-branes, while the length of the total circulation fixes the product of the number of D1 and D5 branes. In other words, once we have set the parametrization in the bubbling ansatz, we have fixed the number of D1 and D5 branes in the system.

The supergravity chiral primaries were studied in [16 − 18]. Among them, there is a special family with $j = \bar{j} = 1, 3/2, \cdots$, that produces fluctuations on the metric of the 3-sphere, associated with the anti-self dual part of the 3-form $H_{(3)}$. In [19] it was noticed that such states are related to changes on the shape of the profile **F**. Since this is the only freedom left in our supergravity solutions, we conclude that these are precisely the chiral primaries that we can probe or excite within the bubbling framework.

Following Mathur et al [20], we use the twist operators $(\sigma_n^{++}, \sigma_n^{+-}, \sigma_n^{--}, \sigma_n^{-+})$[4] to describe the chiral primaries under study. Here σ_n permutes cyclically the ends of each D-string in the system. Since we have a total $N = Q_1 Q_5$ of such strings, n runs from 1 to N. Due to the fact that we consider the case $j = \bar{j}$, we are allowed to use only $(\sigma_n^{++}, \sigma_n^{--})$. Also, σ_1^{--} has the lower conformal dimension, therefore the vacuum configuration in the twist sector is given by the operator $[\sigma_1^{--}]^N$, while a general configuration is given by the product of $[\sigma_{n_i}^{--}]^{m_i}$ and $[\sigma_{n_j}^{++}]^{m_j}$ subject to the constraint $\sum(n_i m_i + n_j m_j) = N$.

Let us consider the vacuum configuration. First we recall that the vacuum maximizes the total spin (since we have N aligned spin 1/2 short strings). From the bubbling point of view, the vacuum also maximizes angular momentum or, better, corresponds to the curve of fixed length that maximizes its angular momentum: the circle. In detail, we find the vacuum by considering first a circular profile

$$\begin{aligned} \vec{F} &= a\cos(wv)\hat{e}_1 + a\sin(wv)\hat{e}_2 \\ |\dot{\vec{F}}| &= aw \end{aligned}$$

[4]These operators expand a $(1/2, 1/2)$ representation of $SU(2) \times SU(2)$. For more details see the original paper.

where $(\hat{e}_1, \hat{e}_2)$ are the unit vectors on the two dimensional plane defined by (x^1, x^2). Then, using the two constraints $f = 4\pi^2\sqrt{Q_1Q_5}$, and $|\vec{F}| = \sqrt{Q_1/Q_5}$, we obtain that $a = \sqrt{Q_1Q_5}$ and $w = 2\pi/l$. Inserting the resulting curve into equation (2.3), we get

$$h^2 = \frac{a}{\sqrt{(y^2+r^2+a^2)^2 - 4a^2r^2}}\,, \qquad z = \frac{1}{2}\frac{y^2+r^2-a^2}{\sqrt{(y^2+r^2+a^2)^2-4a^2r^2}}$$

where $\delta_{ij}dx^idx^j = dr^2 + r^2 d\psi$. Using the following change of coordinates $y = a\,\sigma\sin\theta$, $r = a\sqrt{\sigma^2+1}\,\cos\theta$, we recover $AdS_3 \times S^3$ metric in global coordinates

$$ds^2 = L^2[-(1+\sigma^2)dt^2 + \frac{1}{1+\sigma^2}d\sigma^2 + \sigma^2 d\theta_1^2] + L^2\left(d\theta^2 + \cos^2\theta d\psi^2 + \sin^2\theta d\theta_2^2\right),$$

with $L = (Q_1Q_5)^{1/4}$.

The other celebrated example is the pp-wave solution that, as in the case of bubbling for D3-branes, can be obtained by focusing on local part of the curve **C**, which looks like a straight infinite line. In this limit, we get

$$h^2 = \frac{1}{2\sqrt{(y^2+x_2^2)}}\,, \qquad z = \frac{x_2}{2\sqrt{(y^2+x_2^2)}}\,,$$

where (x_1, x_2) are adapted coordinates, such that they are perpendicular and parallel respectively to the curve **C**. Then, changing coordinates like in (2.2), we obtain the familiar metric

$$ds^2 = -(r_1^2+r_2^2)dt^2 - 2dtdx_1 + dr_1^2 + dr_2^2 + r_1^2 d\theta_2 + r_2^2 d\theta_1\,.$$

Once we have set the dictionary for the vacuum, we can start to study excited states on this sector of the theory. We point out that we can very well consider disconnected curves like those of figure 3, remaining on minimal supergravity domains. Hence, a little puzzle comes into our minds: these configurations seem to be related to giant gravitons, but the latter were associated to different supergravity solutions with non-trivial dilaton behaviour in [13], and therefore should be out of the minimal supergravity theory. To solve this puzzle more studies on the disconnected curves need to be done, that are left for future works.

4. Bubbling and non-regular solutions

In previous sections, we set the rules for bubbling in the D1/D5 system, giving also the dual CFT chiral operators that can be probed. We could now research on physical implications of this sector of the theory. Nevertheless, we decided to postpone such a study for better times, and to work out, instead, the appearance (and the CFT description, if any) of non-regular solutions. Therefore, in this section we will be working with a slight generalization of the bubbling ansatz, relaxing the regularity conditions. In this form, we are able to recover other sectors on the CFT theory (not included into the regular bubbling ansatz that nevertheless is interesting in its own rights) containing for example conical defect metrics.

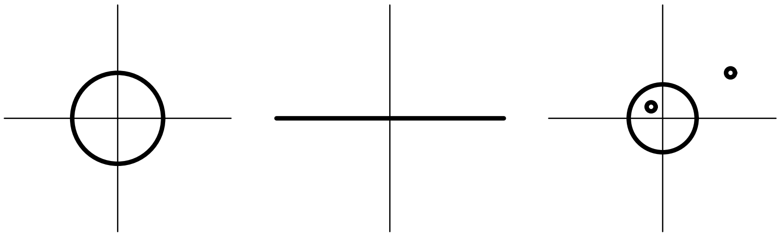

Figure 3: Three possible configurations of bubbling: (a) the vacuum geometry $AdS_3 \times S^3$, (b) its Penrose limit, corresponding to the pp-wave solution, and (c) a superposition of disconnected closed curves in and out of the main big circle, that may be related to giant graviton configurations in S^3 and AdS_3 respectively. In fact, this is the type of figures we get if we write giant graviton test branes in bubbling coordinates, replacing the small circles by points.

From the D1/D5 brane perspective, among the semiclassical solutions, there is some room for generalization: we can relax the constraints on $\mathbf{F}$, by allowing either $\dot{\vec{F}} = 0$ along the curve or self intersections. Also we can consider superposition of regular profiles. On the other hand, from the perspective of bubbling, we have a lot of room to play with. Basically, as soon as we give up the regularity conditions, we have two independent sources. Here, we will conserve the notion of *lines of charge as sources*, and work with asymptotically $AdS_3 \times S^3$ solutions and with round profiles such that we gain an extra Killing direction that simplifies the studies.[5] We have chosen to parameterize the four possible cases in terms of $2\pi\rho_0$ and $\Delta_- z$, where ρ_0 is the source density for h^2 and $\Delta_\pm z = z_I \pm z_{II}$ ($z_I = 1/2$ from the asymptotic boundary conditions). Then we have

$$\begin{aligned}
&1)\Delta_- z = 2\pi\rho_0 \neq 1\,, && && \\
&2)\Delta_- z = 1 && \text{and} && 2\pi\rho_0 \neq 1\,, \\
&3)\Delta_- z \neq 1 && \text{and} && 2\pi\rho_0 = 1\,, \\
&4)\Delta_- z \neq 1\,,\ 2\pi\rho_0 \neq 1 && \text{and} && \Delta z \neq 2\pi\rho_0\,.
\end{aligned} \tag{4.1}$$

while the regular case is recovered by setting $\Delta_- z = 2\pi\rho_0 = 1$.

The fact that we are considering only circular profiles of radius a reduces the form of z, h^2 and V_ψ (the additional Killing vector sets $V_r = 0$) to

$$\begin{aligned}
z(r,y) &= \frac{1}{2}\left[\Delta_+ z + \frac{\Delta_- z(y^2 + r^2 - a^2)}{\sqrt{(y^2 + r^2 + a^2)^2 - 4r^2a^2}}\right]\,, \\
h^2(r,y) &= \frac{L^2}{\sqrt{(y^2 + r^2 + a^2)^2 - 4r^2a^2}}\,, \\
V_\psi &= \frac{1}{2}\Delta_- z\left[1 - \frac{R^2 + y^2 + a^2}{\sqrt{(R^2 + y^2 + a^2)^2 - 4R^2a^2}}\right]\,,
\end{aligned}$$

[5]This is a sort of minimal generalization, but obviously other more general deviations from the regular conditions could be considered. Here we are just probing a reduced region of the space of possible solutions.

where $L^2 = \sqrt{Q_1 Q_5}$. Since we work with a total fixed flux $f = 4\pi^2\sqrt{Q_1 Q_5}$, the radius of the circular profile is constrained to $a = L^2/2\pi\rho_0$. We also change to AdS-adapted coordinates

$$y = L^2\sigma\sin\theta\,, \qquad r = l^2\sqrt{\sigma^2+\alpha}\cos\theta$$

where $\alpha = 1/(2\pi\rho_0)^2$, obtaining

$$\begin{aligned} z(\sigma,\theta) &= \frac{1}{2}\left[\Delta_+ z + \Delta_- z\frac{(\sigma^2-\alpha\sin^2\theta)}{(\sigma^2+\alpha\sin^2\theta)}\right], \\ h^2(\sigma,\theta) &= \frac{1}{L^2(\sigma^2+\alpha\sin^2\theta))}, \\ V_\phi &= -\frac{\alpha\Delta_- z\cos^2\theta}{(\sigma^2+\alpha\sin^2\theta)} \end{aligned} \tag{4.2}$$

and since $z_I = 1/2$, only one of the two $\Delta_\pm z$ is independent, rendering the whole supergravity solution a function of only $\Delta_- z$ and α.

We would like to identify which of the above supergravity solutions are *physical solutions*, that is to say which ones have a CFT dual configuration. At first sight, it seems that we are out of the D1/D5 system: after all, there should be only one source in the physical situation (here, in general, we are dealing with two independent sources). Nevertheless, we know that there are, within the bubbling ansatz, non-regular solutions that correspond to condensates of otherwise regular solutions with CFT dual states.[6]

In the D1/D5 system we could change the value of $\Delta_- z$ trying to see this as the result of an *average* over delocalized sources. The above procedure could also be implemented on the boundary, changing the local amount of D1-branes we locate on this curve, that therefore varies the value of $2\pi\rho_0$. Hence, we just can not rule out the family of solutions (4.2) without further studies. Nevertheless, certainly there are ranges of the parameters $\Delta_- z$ and $2\pi\rho_0$ that do not have any associated CFT dual configuration. Hopefully, the above non-physical solutions should be associated to CTC or other types of pathological behaviours.

In fact, one of the general features of the solutions (4.2), is that there are CTC. Take for example the $g_{\psi\psi}$ components of the metric

$$g_{\psi\psi} = L^2\frac{\cos^2\theta}{\sigma^2+\alpha\sin^2\theta}\left[\sigma^2+\alpha\left(1-\alpha(\Delta_- z)^2\cos^2\theta\right)\right]$$

from which we can see that there are regions in space-time with CTC if

$$\alpha > (\Delta_- z)^{-2} \qquad \text{or if you prefer} \qquad |\Delta_- z| > |2\pi\rho_0|\,.$$

These regions are defined by the equation

$$\sigma^2 < \alpha[\alpha(\Delta_- z)^2\cos^2\theta - 1]$$

and the metric has a naked singularity for $\theta = \sigma = 0$ (see figure 4).

[6] For example, in the D3-brane case we have the superstar solution [21], that can be understood as a sort of average over the metrics produced by the giant graviton distribution [22]. In this case, we relax the two possible boundary values of z associated to white and black in the figures, such that we also have grey tonalities, associated to regions where the average is considered and therefore the source is effectively smeared.

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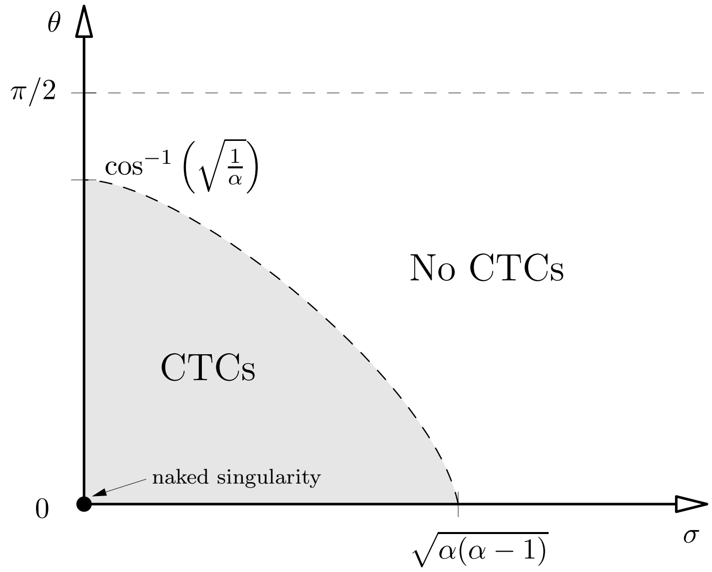

Figure 4: The picture shows the CTC region for fixed α in the (θ, σ) plane.

It would be very interesting to translate the above supergravity range of parameters into CFT parameters, to actually understand if such pathological solutions are or not in the CFT theory. Unfortunately, these supergravity solutions would be related to averages over microscopical configurations, and it is not assured that we will discover how the average was done, and over which microstates. Nevertheless, we have been able to find the CFT dual configuration for the first two cases listed in (4.1), respectively corresponding to conical singular metrics and to the Aichelburg-Sexl type metric found in [14]. In both cases, we found CFT duals only for the non pathological regimes, enforcing a protection mechanism curing gravity.

Case $\Delta_{-}z = 2\pi\rho_0 \neq 1$. Let us consider the first case, i.e. where $\Delta_{-}z = 2\pi\rho_0 \neq 1$. We have chosen to parametrize it as follows,

$$\alpha = (1+Q)^2\,, \qquad \Delta_{-}z = \frac{1}{(1+Q)}\,,$$

where, in general, Q runs from $-\infty$ to ∞. The resulting form of the metric is

$$\begin{aligned} ds^2 \,=\, & L^2[-((1+Q)^2+\sigma^2)dt^2 + \frac{d\sigma^2}{((1+Q)^2+\sigma^2)} + \sigma^2 d\bar{\theta}_1^2 + \\ & +d\theta^2 + \cos^2\theta d\bar{\psi}^2 + \sin^2\theta d\bar{\theta}_2^2] \end{aligned}$$

where

$$\bar{\theta}_1 = \theta_1\,, \qquad \bar{\theta}_2 = \theta_2 + Q\theta_1\,, \qquad \bar{\psi} = t + \psi$$

This metric corresponds to a local $AdS_3 \times S^3$ space-time, with a conical singularity. If $0 < (1+Q) < 1$ we have a conical defect, but if $1 < (1+Q) < 2$ we have a conical excess.[7]

[7]Other values of Q are irrelevant, due to the periodicity of $\bar{\psi}$.

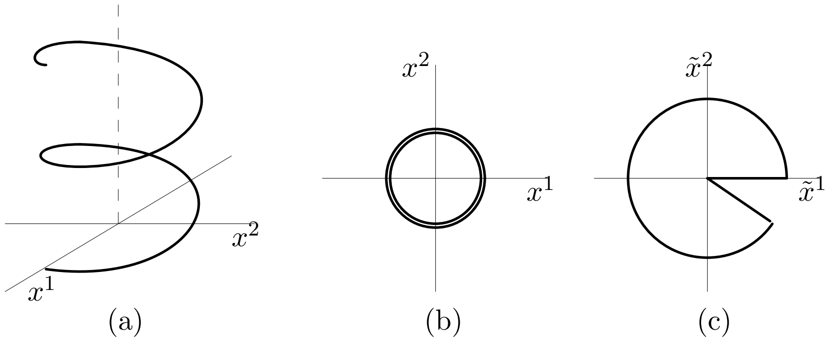

Figure 5: (a) shows a profile that circulates more than once with frequency w_Q. (b) shows its projection into the (x^1, x^2)-plane, where the double line is stressing the self interaction of the curve. (c) shows the same profile in terms of new variables $\tilde{x}^i = \frac{1}{(1+Q)} x^i$. In this new frame, the radius of the circle is L^2 as in the vacumm case but the curve fails to close, with deficit angle $\delta\bar{\theta}_1 = 2\pi Q$.

To make contact with the D1/D5 system, we have to set the correct parametrization of the curve $\mathbf{C}$. We found that the form of the curve and the flux constraint define uniquely the parametrization to be given by

$$\begin{aligned} \vec{F} &= a\cos(w_Q v)\hat{e}_1 + a\sin(w_Q v)\hat{e}_2\,, \qquad & v &= (0,l) \\ a &= \sqrt{Q_1 Q_5}(1+Q)\,, & w_Q &= \frac{2\pi}{l(1+Q)} \end{aligned}$$

At this point, we have to introduce the physical constraint that the curve $\mathbf{F}$ is actually closed. This is only achieved if $Q = (1-m)/m$ with $m \in \mathbb{N}_0$. In other words, we can find a corresponding configuration on the CFT side, as long as $Q \in (-1, 0)$. These allowed cases produce only conical defect metrics, with deficit angle $\delta\bar{\theta}_1 = 2\pi Q$. Of course this case is not regular since we have self intersections on $\mathbf{F}$.

The relation between conical defects and CFT operators has been studied before for the D1/D5 system [14]. The dual associated operator is given by

$$[\sigma_m^{--}]^{N/m}$$

and produces a deficit angle $\delta\bar{\theta}_1 = 2\pi(1 - 1/m)$. Notice that the system is excited to higher energy state that in the U-dual P/F1 picture corresponds to the harmonic $w_m = 2\pi m/l$.

So, we have seen how to recover conical defect metrics within the bubbling picture, in a completely independent way, compared to how such metrics were found in the D1/D5 system. Also, we have gained more physical input into other related supergravity solutions, the conical excess. From the above considerations, these solutions are not in the spectrum of the dual CFT theory and, therefore, should be ruled out as un-physical solutions.[8] This is a typical behaviour of string theory, which teaches

[8]One way to observe the impossibility of such states is by noticing that they imply the existence of harmonics of lower energy than $w_1 = 2\pi/l$, which is the ground energy level, and corresponds to an initial working assumption. All the above is in the U-dual P/F1 system.

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us that not all the supergravity solutions are to be labeled as physical (keeping in mind that gravity is just an effective theory). In this particular example, the result is not so unexpected, since conical excess can be thought as the response of space-time to negative energy point particles, that even at the classical level are somehow related to pathologic behaviours. Nevertheless, this example shows the power of the AdS/CFT duality since it leaves no room for doubts about the physical meaning of these metrics.

Case $\Delta_- z = 1\ ,\ 2\pi\rho_0 \neq 1\,.$ In this second case we set the parametrization as follows

$$\alpha = (1+Q)\,, \qquad \Delta_- z = 1\,,$$

where Q runs from -1 to ∞. The resulting metric is

$$\begin{aligned} ds^2 = & -L^2(\sigma^2+\alpha)dt^2 + \frac{L^2}{\sigma^2+\alpha}d\sigma^2 + L^2\sigma^2\frac{\sigma^2+\alpha\sin^2\theta}{\sigma^2+\alpha^2\sin^2\theta}d\theta_1^2 + \\ & +L^2 d\theta^2 + L^2\sin^2\theta\frac{\sigma^2+\alpha^2\sin^2\theta}{\sigma^2+\alpha\sin^2\theta}\left(d\theta_2^2 - \frac{(1-\alpha)\sigma^2}{\sigma^2+\alpha^2\sin^2\theta}d\theta_1^2\right)^2 + \\ & +L^2\alpha\cos^2\theta d\tilde{\phi}^2 + \frac{L^2\cos^2\theta}{\sigma^2+\alpha\sin^2\theta}(1-\alpha)(\sigma^2+\alpha^2)(d\tilde{\phi}-dt)^2 \end{aligned} \tag{4.3}$$

that corresponds to the Aichelburg-Sexl metric found in [14]. This can be easily seen by comparing the form of (h^2, V_ψ) with the corresponding metric functions given in [13] (see equations (3.18) (3.20) and use $q = -Q$). These solutions were found in the D1/D5 framework by smearing over a single turn on the circular profile $\mathbf{F}$ a large number of *bits of the curve* that remain at a fixed point in space-time, while we move on the curve of parameter v. Therefore we are changing the density of D1-branes in the resulting average curve (see [13] for a complete construction of this solution).

The above metrics are conjectured to be dual to operators of the form

$$[\sigma_1^{--}]^p\sigma_{n_1}^{--}\sigma_{n_2}^{--}\ldots\sigma_{n_k}^{--}$$

with $n_i \gg 1$. The n_i's correspond to *bits* on the U-dual F-string profile, that remain constant at a fixed point in space-time, while we move on the parameter v along the curve. The average on the position corresponds to a distribution with a large dispersion on n_i. Then, large numbers of such fixed *bits* will translate into a reduction of the radius a of the circle described by the profile $\mathbf{F}$, since less *bits* will be left to close the curve. In the above sense, radii larger than the radius of the vacuum configuration are impossible to be constructed in the CFT theory. Therefore we conclude that the solutions with $Q \in (-1,0)$ (or if you prefer $\alpha > 1$) are not physical, since there is no dual CFT configuration. At this point, we stress that precisely when $\alpha > 1$, the supergravity solutions present CTC! Therefore this is another example of chronology protection implemented by string theory[9].

[9] Also, it is interesting to notice that in the asymptotic regions of large σ, these metrics (4.3) suffer from conical alike defect or excess, depending on whether α is less or bigger than one, reinforcing our previous conclusions.

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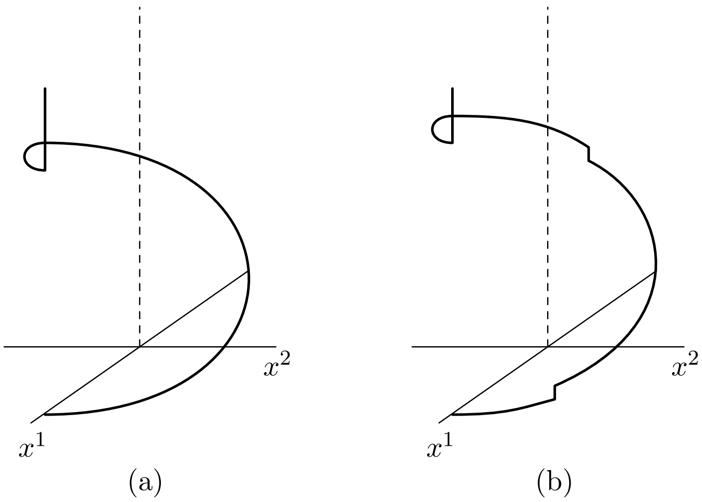

Figure 6: (a) shows a curve that circulates once around the origin, with zero velocity in the last portion of the curve. (b) shows a curve that has many parts whith zero velocity. The Aichelburg-Sexl metric is obtained by the limiting situation when the straight bits are smeared along the curve.

5. Summary and discussion

In this article we have studied the relation between bubbling in AdS_3 and the more conventional construction of microstates in the D1/D5 system. We have learned a few lessons: the first lesson tells us how both constructions are related once we concentrate on regular solutions only. In this case the D1/D5 family of microstates contains the bubbling solutions as a subset. The connection is possible due to the collapse (in the bubbling framework) of the different sources into a single line of charge. This subset is dual to a particular tower of chiral primaries operators in the CFT with conformal weights $(1, 3/2, \ldots)$. Therefore, we have studied all the geometries sourced by connected and/or disconnected closed curves with fixed total length. The second lesson tells us that this is not the full story and that non-regular solutions have also a role to play in this framework. This time, the splitting of the sources (characteristic of the bubbling picture) is understood (from the D1/D5 family of microscopic states) as the result of an average over semiclassical configurations, that effectively smears the string source. The third lesson is that there are solutions within the bubbling ansatz that have no counterpart in the D1/D5 family of microstates and, therefore, do not have a CFT dual. These solutions are artifacts of the low energy supergravity theory and should be discarded as non-physical. In particular, in all the non-physical solutions we have found there are pathologies like CTC. Hence string theory seems to be acting as a chronology protection agency.

It would be very interesting to connect the AdS/CFT picture with the Liouville theory living at the boundary of AdS_3. In fact, it was shown in [23] that the world-volume theory of a single D1-brane in AdS_3 becomes precisely a Liouville theory once we approach the boundary. The D1-brane needs to be rotating in the S_3 to become a stable BPS state. Such states are called giant gravitons and are of importance in the bubbling framework (at least for the D3-brane case). Now, it is also known that in the Liouville theory there are normalizable and non-normalizable states (see [24] for a review). The normalizable states have a continuous spectrum bounded from below, while the non-normalizable states do not present such a gap. Therefore, the theory gets organized as a series of sectors labeled by this non-normalizable states plus the tower of normalizable states. In [25] non-normalizable states were conjectured to be dual to conical defect metrics. At this point we would like to recall that, from the bubbling point of view, the CFT theory is somehow naturally arranged into regular and non-regular sectors, in such a way that the non-regular sectors act as different vacua, while the regular sector can be accommodated as a deformation on each of these vacua. In terms of bubbling figures we have, for example, a conical defect vacuum, defined by a circular self-interacting curve, and a whole tower of operators, produced by small changes on this profile, deforming the circle. Each conical defect metric is defined by a winding number that separates one sector from the other. So we believe that these similarities signal common structure and that a deeper study on giant gravitons on the D1/D5 system deserves attention, since it may provide a bridge between dual CFT theory of D1/D5 system and the Liouville theory at the boundary of AdS_3.

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